Can Agglomerations Self-destruct? Crowding of the Product Markets as a Dispersion Force in Economic Geography Models

Ali lahko pride do samouničenja aglomeracij? "Zgostitve" na izdelčnih trgih kot disperzijska sila v modelih ekonomske geografije

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Abstract

This paper develops an approach to analysing the possible effects of product market crowding on the adjustment of manufacturing activity in economic geography models. With the aim of analysing the role of crowded product markets in determining the location of manufacturing I augment the celebrated Krugman-Venables models by combining a perfectly mobile manufacturing labor force with the inclusion of intermediate goods in production. In order to test the effects of a crowded product market I also derive and test several functional relationships between the number of firms competing in the market and the elasticity of substitution. Crowding of the product markets reveals itself as a strong dispersion force effectively counteracting the agglomerating effects of a progressively crowding product space and high transport costs enable the dispersion of economic activity to occur despite a perfectly mobile labor force and the existence of intermediate goods markets.

Keywords: Elasticity of substitution, Product space crowding, Economic geography, Agglomeration, Dispersion

JEL classification: L11, F12, R12

Povzetek

V pričujočem prispevku razvijam pristop h analizi učinkov povečevanja konkurence na trgih posameznih izdelkov na prostorsko dinamiko ekonomske aktivnosti v modelih ekonomske geografije. Z namenom analize učinka "gostih" izdelčnih trgov pri določanju lokacije predelovalne industrije sem bil primoran znatno predelati Krugman-Venablesov model ekonomske geografije, tako da sem vanj vpeljal mobilnost delovne sile in proizvodnjo vmesnih proizvodov. Pri testiranju učinkov zaostrovanja konkurence na izdelčnih trgih sem razvil več konkurenčnih specifikacij funkcijske povezave med številom konkurenčnih podjetij in elastičnostjo substitucije. Izkaže se, da povečevanje konkurence na trgih posameznih izdelkov deluje kot močna disperzijska sila, ki deluje v nasprotju z aglomeracijskimi pritiski mobilne delovne sile in uporabe vmesnih proizvodov.

1 Introduction

Models of new economic geography or spatial economics attempt to theoretically formalize the empirically observed phenomenon of geographical concentration or dispersion of certain industries by constructing general equilibrium models with explicitly included transport costs. These models commonly assume a constant elasticity of substitution independent of the number of varieties in the market, which allows for useful simplifications, but also serves to exclude a possibly important force from the model.

My aim in the following disposition is to highlight the importance of the above assumption for the implications of economic geography models. In abandoning the constant elasticity of substitution assumption I specify several different functional forms that could serve to describe the relationship between the elasticity of substitution and the number of available varieties. Given these specifications I test an economic geography model that includes intermediate goods as well as a perfectly mobile manufacturing labour force. Different specifications of the elasticity functions turn out to effect the allocation of economic activity significantly, which casts serious doubts on the validity of the assumption of a constant elasticity of substitution. As is turns out, depending on the functional form assumed for the elasticity specification, endogenous elasticity of substitution is a dispersion factor, forcing firms in agglomerated markets to lower their price cost mark-ups and subsequently relocate.

In the first section I present a brief discussion on the microeconomic foundations that could lead to the occurrence of product space crowding in the growing markets. An economic geography model encompassing both intermediate goods and perfect spatial labour mobility will be presented in the second section, while in the third section I give evidence on the existence of equilibria when product space crowding is explicitly included in such a model. In the final section I offer some possible conclusions based on the approach taken in this paper.

2 Product space crowding

The assumption of a fixed elasticity of substitution between manufacturing varieties has been somewhat of a mainstay in economic geography models based on the representative consumer version of monopolistic competition (Dixit-Stiglitz, 1977) model. Given the importance of the elasticity of substitution for economic geography models the revision of the above assumption could potentially offer new insights into the agglomeration and dispersion dynamics behind the economic geography phenomenon.

The representative consumer maximizes her utility based on the preference for variety of the Spence-Dixit-Stiglitz form (Spence, 1976, Dixit-Stiglitz, 1977), whereby she ends up consuming all the available varieties with relative prices determining the relative quantities for given variety pairs. Additional varieties enter the utility function symmetrically thereby increasing her overall utility.¹ The assumption of constant elasticity of substitution although significantly sim-

¹The preference structure implemented here is also known as the representative consumer approach as opposed to the address approach of Lancaster (1966, 1975). Although the address approach would enable simpler modelling of the effects of new firms entering the market it is considered less tractable in general equilibrium modelling. For a very detailed discussion on modelling monopolistic markets see Anderson, de Palma and Thisse (1992)

plifying for the overall analysis of agglomeration and dispersion forces certainly does not add to the credibility of the models by not allowing for the possibility of the increasing number of varieties subsequently crowding the product space and hence decreasing the perceived differences between varieties. Constant elasticity of substitution, in other words, means that the product space broadens each time a new variety is added or, conversely, each time a new firm enters a given market, due to increasing returns to scale in production.² No matter how many new varieties enter the product space it never gets crowded in the perception of the consumer. Each of the competing firms in the market is therefore enabled to maintain its initial level of "monopoly power" for a given variety despite new entrants potentially crowding the product space with more or less similar varieties.

In augmenting the above assumption I will, on the other hand, assume that with each new entry into the market the elasticity of substitution between varieties increases due to the crowding of the market space.³ With each additional variety in the market a consumer is increasingly less able to tell them apart, which increases her perceived elasticity of substitution between them and ultimately decreases the single variety producer's mark-up (and his monopoly power). I

²Given that all varieties are positioned symmetrically in a product space with the distance (elasticity of substitution) between any two varieties being the same. Since two varieties could be positioned symmetrically in such a way in one-dimensional space, e.g. on a line, and three varieties in two-dimensional space, e.g. on a circumference of a circle while four varieties could exists in such perceived symmetry in three-dimensional space, e.g. on a sphere, for n varieties a (n-1)-dimensional sphere would be required. The broadening of the product space in this sense therefore leads to an increase in the dimensionality of the variety space.

³Rumbaugh believes it is reasonable to assume that as the number of varieties increases these become closer substitutes approaching perfect substitutability in the limit (Rombaugh, 1991: 26-8).

therefore propose that the product space would subsequently become crowded with new entries into the market (the dimensions of the product space would still have to increase in order to maintain the symmetric relative position of varieties in the market but the perceived distance between varieties as manifested by the elasticity of substitution would decrease with new varieties entering). The consumer's ability to differentiate between the varieties is, hence, progressively diminishing with the increased dimensionality of the product space, which could be represented by the decreasing distance between varieties in the product space (but a higher dimensional product space⁴). New varieties still enter the market symmetrically, but there is a shift among the existing varieties drawing them closer together in addition to the increase in the dimensionality of the product space. Thus, the elasticity of substitution could be represented by an increasing function of the number of varieties available in the market, which is line with the assumptions made in Dixit-Stiglitz (1977), Laurence-Spiller (1983) and Rumbaugh (1991).⁵

The negative externality represented by product market crowding plays a crucial role in the dynamics of the spatial economy models. Namely, spatially agglomerated economic activity supports the existence of a larger number of firms compared with the more dispersed markets due to the beneficial backward and forward linkages between firms at a given location. The negative effects of a

⁴This can be represented easily in n-dimensional product space, where $n \ge 2$. (e.g. smaller radius of the circle in two-dimensional space, smaller radius of a n-dimensional sphere)

⁵Rumbaugh (1991) notices that increased similarity between varieties of intermediate goods decreases their price as well as that this effect is more pronounced when a smaller number of varieties exists in the market. Feenstra (1994, 2003) states a credible case for the translog expenditure function, which enables the modeling of variability in the elasticity of substitution.

crowded product space in an agglomerated market could force a reconsideration of the costs and benefits involved with operations in the larger market that could befall a potential entrant.

The aim of this paper is to state a case for several versions of the required functional relationship between the elasticity of substitution and the number of firms and then test their effects on the form of convergence in economic geography models. This two stage process should enable me to isolate the possible effects crowded product markets may have on the location of economic activity.

The effect of an increase in the number of firms on the elasticity of substitution is derived by combining profit maximization of producers with the zero-profit condition and cost minimizing behavior of consumers. This approach hence utilizes the monopoly pricing equation (stemming from the profit maximizing behavior of monopolistic producers), which is combined with the cost minimizing price index and the zero-profit condition to give the most general specification of product space crowding ⁶

$$\frac{\partial \rho}{\partial n} = \frac{\frac{(\rho - \alpha)(\rho - n)}{n} + \rho \frac{(\rho - 1)}{(1 - n)}}{\frac{\rho^2 n - \rho^2 + \alpha \rho^2 - \alpha \rho n - \alpha \ln n(\rho - n)(1 - \rho)}{\rho(1 - \rho)}},\tag{1}$$

where n is the total number of firms producing differentiated goods, α is the share of intermediate goods in production and ρ the intensity of the preference

 $^{^{6}}$ The derivation of the obtained functional forms and necessary conditions for a positive effect of the number of varieties on the intensity of preference for variety are given in Appendix B.

for variety.⁷

The increase in σ (or ρ) causes varieties to become closer substitutes and hence decreases the desire of consumers to consume a greater variety of manufactured goods. It is also important to note that a transition from autarky to free trade, which would increase the availability and subsequent consumption of varieties by allowing new varieties into the market, would also increase the elasticity of substitution between these varieties (decreasing the utility increment of an additional variety). Trade liberalization can therefore, under specific circumstances, have an adverse effect on consumer welfare (Montagna, 1998)

On the other hand new entrants also increase competition in the markets, which tends to lower the mark-up of all firms in those markets (through the effect on the demand elasticity). New entrants would therefore have a negative effect on the incumbent producers' price by lowering the elasticity of substitution (the last term in equation (2) is decreasing with increases in the number of varieties on offer⁸) therefore lessening the incentive for agglomeration. Equation (2) by and large stays true to the monopoly pricing equation in Fujita, Krugman, Venables, 1999: 51, but since individual firms now affect aggregate quantities the elasticity of demand (ε) no longer equals the elasticity of substitution (σ). The other important difference compared to the basic monopoly pricing equation is the inclusion of intermediate goods (cost of which is denoted by the price index

⁷The intensity of the preference for variety (ρ) and the elasticity of substitution (σ) are related through $\rho \equiv (\sigma - 1)/\sigma$. For simplicity, I will assume that σ takes on the same value for both consumers and producers.

 $^{^{8}}$ Due to the limitations placed on the length of this paper I will not present derivations of the general functions used in EG models but will instead use the functional forms as derived elsewhere (for further reading see Fujita, Krugman, Venables, 1999).

 G_r).

$$p_r^M = c(w_r^{1-\alpha}G_r^{\alpha})\frac{\varepsilon}{\varepsilon+1},\tag{2}$$

where p_r is price of manufacturing goods in region r, w_r is the nominal manufacturing wage in region r and G_r is the price index of manufacturing goods in region r and ε is the price elasticity of demand. The optimal quantity q^* produced remains equal across locations (because it is defined by technological factors and the elasticity of demand) but now becomes affected by the number of firms through the elasticity of demand (elasticity of substitution⁹)

$$q^* = \frac{F(\varepsilon + 1)}{c}.$$
(3)

The zero-profit condition ensures that an increase in the equilibrium output of a firm is always accompanied by a decrease in the price of that product (this result is in line with Grossmann, 2001)¹⁰. All firms supplying the manufacturing goods move along the demand curve to a more elastic part of the curve. In the limit with the number of varieties increasing the endogenous elasticity of substitution as presented leads to perfect competition with $\sigma = \infty$ and p = MC when the product space is occupied by ∞ varieties.

⁹In contrast with the assumption of a large number of firms in the market, which is commonly used in Dixit-Stiglitz type models of monoplistic competition, no such simplification can be addopted here. The price elasticity of demand (ε) is no longer equal to the elasticity of substitution in absolute value (σ). It turns out that $\varepsilon = -\sigma + \frac{\sigma - 1}{n}$ (for details see Appendix A). One should also note that the demand elasticity depends on the number of varieties available across all locations ($n = \sum_{r=1}^{R} n_r$). ¹⁰Grossmann partly attributes the increased firm size to an increase in fixed costs due to

¹⁰Grossmann partly attributes the increased firm size to an increase in fixed costs due to the competition of firms for consumer attention (increased marketing costs).

3 Spatial model with intermediate goods and perfect labor mobility

In the following section I propose a model of economic geography with intermediate goods in production and a perfectly mobile manufacturing labour force. I will maintain the model's structure originally set up by Krugman (1991) and Krugman-Venables (1995, 1996) and reintroduced in a uniform framework in Fujita, Krugman, Venables (1999), but will abandon the assumption of infinitely expanding product markets introducing the possibility of product space crowding discussed in the previous section.

Consumer behavior in this model does not change from the one assumed in Fujita, Krugman, Venables (henceforth FKV), 1999. I therefore maintain the framework of the representative consumer maximizing her utility by choosing the consumption of agricultural or manufacturing products satisfying her upperlevel utility, but subsequently also deciding on the consumption of manufacturing varieties at the lower-level consumption decision¹¹.

4 Model

I assume that the world consists of two regions, both capable of producing manufacturing and agricultural goods. The world endowment of manufacturing workers is split between both regions into shares λ for region 1 and (1- λ) for region 2, while agricultural workers are split into shares ϕ and (1- ϕ) respectively.

¹¹For further details see FKV, 1999: 46-9.

Units of measurement are chosen so that the total number of workers in the world can be split into shares δ of manufacturing workers and (1- δ) of agricultural workers.

Manufacturing workers are assumed to be mobile between regions in response to differences in real wages, while agricultural workers are not mobile between regions. There is no labour mobility between the two sectors.

Agriculture functions under constant returns to scale with no transport costs incurring to agricultural goods in transport between regions. This leads to an equalization of agricultural wages across the two regions, which are set to equal 1 (numeraire wage).

Income in a given region is hence determined by

$$Y_1 = \delta \lambda w_1 + (1 - \delta)\phi \qquad Y_2 = \delta (1 - \lambda) w_2 + (1 - \delta)(1 - \phi).$$
(4)

Since share $(1-\alpha)$ of total revenue is spent on wages (FKV, pg. 243)

$$w_r \lambda_r = (1 - \alpha) n_r p_r q^*, \tag{5}$$

using (2) and (3) it follows that

$$w_r \lambda_r = (1 - \alpha) F n_r w_r^{1 - \alpha} G_r^{\alpha} \varepsilon \qquad and \qquad n_r = \left(\frac{w_r}{G_r}\right)^{\alpha} \frac{\lambda_r}{(1 - \alpha) F \varepsilon}, \qquad (6)$$

where n_r is the number of varieties produced in region r and F the fixed costs of manufacturing good production. Given that there are only two locations to consider and assuming that the adjustment of σ to the number of firms in the market of a given location is instantaneous, then (7) and (8) apply

$$n_1 = \left(\frac{w_1}{G_1}\right)^{\alpha} \frac{\lambda}{(1-\alpha)F\varepsilon},\tag{7}$$

$$n_2 = \left(\frac{w_1}{G_1}\right)^{\alpha} \frac{1-\lambda}{(1-\alpha)F\varepsilon},\tag{8}$$

where λ is the share of manufacturing labour force in region 1 and the total manufacturing labour force in the home country is normalized to 1. The total number of varieties produced is hence $n = n_1 + n_2$.

Equations for the respective price indices become

$$G_{1}^{1-\sigma} = \frac{w_{1}^{1-\sigma(1-\alpha)}G_{1}^{-\alpha\sigma}\lambda c^{1-\sigma}(\varepsilon-1)^{\sigma-1}}{(1-\alpha)F\varepsilon^{\sigma}} + \frac{w_{2}^{1-\sigma(1-\alpha)}G_{2}^{-\alpha\sigma}(1-\lambda)c^{1-\sigma}(\varepsilon-1)^{\sigma-1}T^{1-\sigma}}{(1-\alpha)F\varepsilon^{\sigma}}$$

$$G_{2}^{1-\sigma} = \frac{w_{1}^{1-\sigma(1-\alpha)}G_{1}^{-\alpha\sigma}\lambda c^{1-\sigma}(\varepsilon-1)^{\sigma-1}T^{1-\sigma}}{(1-\alpha)F\varepsilon^{\sigma}} + \frac{w_{2}^{1-\sigma(1-\alpha)}G_{2}^{-\alpha\sigma}(1-\lambda)c^{1-\sigma}(\varepsilon-1)^{\sigma-1}}{(1-\alpha)F\varepsilon^{\sigma}}$$

$$(10)$$

Equations (9) and (10) (as well as (11) and (12)) introduce transport costs as a crucial determinant into the model. As is common practice in economic geography models the modeling of the transport sector is simplified by the adoption of the iceberg assumption. Predictably, the price index (G) turns out to be higher in the location that has to import the larger share of the total number of varieties of manufacturing products. Manufacturing wage equations¹² or the break-even wage turns out to be

$$(w_1^{1-\alpha}G_1^{\alpha})^{\sigma} = \frac{c}{F(\varepsilon-1)} \left[E_1 G_1^{\sigma-1} + E_2 G_2^{\sigma-1} T^{1-\sigma} \right], \tag{11}$$

¹²Real manufacturing wages are defined as $\omega_r = w_r/G_r^{\mu}$ (where μ is the share of manufacturing in consumption).

$$(w_2^{1-\alpha}G_2^{\alpha})^{\sigma} = \frac{c}{F(\varepsilon-1)} \left[E_1 G_1^{\sigma-1} T^{1-\sigma} + E_2 G_2^{\sigma-1} \right],$$
(12)

where E is expenditure on manufactures. Again, the manufacturing wage equations depicted in (11) and (12) stay true in most parts to the original contribution of FKV, 1999, with the only differences resulting from the fact that the elasticity of substitution (σ) is no longer constant and hence the equilibrium quantity produced (q^*) can vary as well.

Expenditure on manufactures (following FKV) is defined as

$$E_1 = \mu Y_1 + \frac{\alpha w_1 \lambda}{1 - \alpha},\tag{13}$$

$$E_2 = \mu Y_2 + \frac{\alpha w_2 (1 - \lambda)}{1 - \alpha}.$$
 (14)

The equations turn out to resemble those in FKV, 1999: 242-3 quite closely with the only differences being caused by the fact that labour mobility was added to the structure of the model and the fact that certain simplifications are prevented by the abandonment of the constant elasticity of substitution assumption.

The above system of equations represents a set of eight equations ((4), (9)-(14))in eight unknowns, not including the elasticity of substitution and the number of varieties, but that subsystem of equations also turns out to be identified.

In the following section I therefore present simulations of the above system, whereby for computational purposes I rely on an iterative process leading to asymptotic values of the unknown variables.

5 The existence of equilibria in the model with

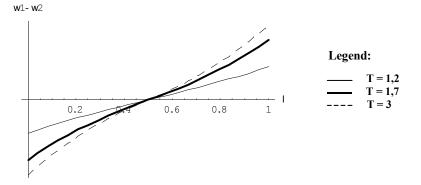
two locations

In this section I investigate the existence of equilibria in the model laid out in the previous section. To estimate the possible effects of commodity space crowding different functional forms of the relationship between elasticity of substitution and the number of varieties in the market are used (see Appendix B) with the emphasis put on the most general specification (1). The simulations to be presented analyze the response of regional real wages to changes in the share of manufacturing labour in region 1. The dynamics of regional real wage responses are, in addition, analyzed at three different levels of trade costs in order to determine the effects of changes in costs of trade on the observed processes. For the sake of brevity I only present the simulations with the two extreme functional forms out of those derived in Appendix B.

Firstly, I present a simulation of the above model with a constant elasticity of substitution. This serves a dual purpose, in addition to outlining the model's properties without the additional complications caused by product space crowding it also serves as a benchmark for analysis of the effects of the alternative specifications of the elasticity function. Figure 1 depicts the regional real wage difference (ω 1- ω 2) response to changes in the share of the manufacturing labour force in region 1 for three different levels of transport cost.

Figure 1: Simulation of the model specification with constant elasticity of

substitution (three different cost specifications)



Predictably, the model proposed in section 4 exhibits very strong tendencies towards agglomeration due to the inclusion of both intermediate goods markets as well as a freely mobile manufacturing labour force in the model. Both factors tend to drive the agglomeration of economic activity in one location at the expense of the other location. The celebrated core-periphery pattern therefore manifests itself regardless of the size of transport costs employed in the simulation. Even with high transport costs (T = 3), as represented by the dashed line in Figure 1, there are only two stable equilibria with all manufacturing concentrated in either of the two regions.

The second set of simulations assumes the simplest, logarithmic functional form of the elasticity equation derived in Appendix B

$$\rho = \frac{\ln n}{\ln k},\tag{15}$$

where k serves as a standardization variable.¹³ Figure 2 presents the regional real wage difference ($\omega 1$ - $\omega 2$) given shares of the manufacturing labour force in region 1 as generated by the above model with the elasticity specification (15) for different levels of transport cost.

Figure 2: Simulated response of the wage difference to changes in the share of

manufacturing labor in region 1 (with different levels of transport costs)

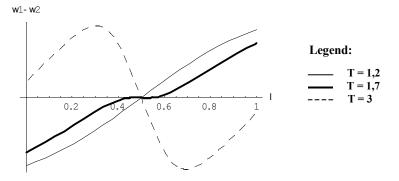
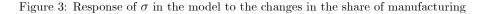
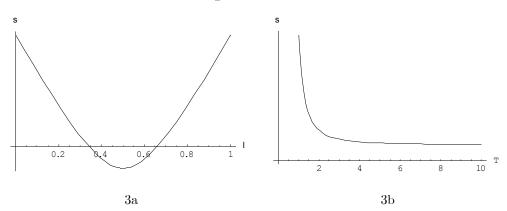


Figure 3 shows the response of the endogenous elasticity of substitution σ to the change in the distribution of manufacturing labour under the logaritmic specification (15) with trade costs set at 1.5 and the response of σ to changes in trade costs with the share of manufacturing labor $\lambda = 0.5$.

¹³Due to the restriction $(0 < \rho < 1)$ on the ρ (15) has to be rewritten in the elasticity form, as well as additional restrictions have to be placed on k in order for this specification to be used in the simulations.





labour and changes in trade trade costs

Figure 2 reaffirms the initial belief that the inclusion of both intermediate goods as well as labour mobility in the model will tend to create strong agglomeration forces that prove to be difficult to counteract by dispersion forces to a sufficient degree so as to prevent a core-periphery pattern from developing. The prevalence of the complete agglomeration outcome is obvious when trade costs are low (T = 1.2) represented by the solid line in Figure 2. In this case highest relative wages for a given location come about when all manufacturing labour is concentrated in that location and none in the other. Not unlike the results presented in FKV (1999) an increase in trade costs changes the form of the wage adjustment curve with medium level trade costs (T = 1.7) leading again to three possible stable equilibria (beside the two complete agglomeration outcomes a symmetric equilibrium is also possible) and two unstable ones. The importance of trade costs gets even more accentuated with higher trade costs (T = 3) represented by the dashed line, where only one of the possible equilibria turns out to be stable. Only the symmetric equilibrium (with labor share split equally between the two regions) prevails with high trade costs¹⁴. Large trade costs between the two locations tend to limit the bilateral trade volumes and ultimately force manufacturing to disperse equally between both regions.

These results seem remarkably similar to those shown in FKV (1999: 66-7), but there are also other crucial considerations to be made when comparing the two outcomes. First of all, the model presented here includes both intermediate goods and a perfectly mobile manufacturing labour force, while the results in FKV rely solely on perfect labour mobility. Secondly, there are important changes to the wage response curves brought forth by the effects of the endogenous elasticity of substitution on the geographical adjustment of the manufacturing labour force. As can be seen from Figure 3a elasticity of substitution increases with agglomeration, which tends to decrease producer prices and mark-ups as well as increase the equilibrium quantities produced (implied by the zero-profit condition). In addition σ tends to decrease with trade costs as seen from Figure 3b (due to a decrease in the number of firms).

Comparing Figure 2 with Figure 1 it becomes obvious that endogenous elasticity of substitution serves as a strong dispersion factor, which helps to mitigate the strength of agglomeration forces (intermediate goods and perfect manufacturing labour mobility)¹⁵. This can be seen by observing that in Figure 1 even with

 $^{^{14}}$ The fact that the derivative of the wage difference with respect to the share of manufacturing labor in region 1 is negative ensures that the symmetric equilibrium is in fact stable.

¹⁵Endogenous elasticity of substitution therefore complements the usual dispersion factors such as factor costs, costs of nontradables and trade costs. This is in contrast with the pure labour mobility model, where a symmetric distribution can be obtained with higher trade costs serving as a dispersion cost (FKV, 1999: 65-68). Trade costs alone do not suffice in this model due to the strength of agglomeration forces.

high trade costs (T = 3), the simulated wage differences do not display any possibilities of a symmetric equilibrium forming and a core-periphery pattern is maintained regardless of the trade costs.

The third set of simulations employs the most general specification of the elasticity function given by

$$\frac{\partial \rho}{\partial n} = \frac{\frac{(\alpha - \rho)(n - \rho)}{n} + \frac{\rho(1 - \rho)}{(n - 1)}}{\frac{n(\rho - \alpha) + \rho(1 - \alpha)}{(1 - \rho)} + \frac{\alpha \ln n[n - \rho]}{\rho}}.$$
(16)

The derivation of the above equation as well as the conditions for a positive effect of the number of varieties on the elasticity of substitution are presented in Appendix B. The allocation of economic activity under the elasticity specification (16) is given in Figure 4.

Figure 4: Simulated response of the wage difference to changes in the share of

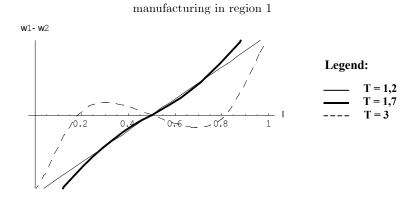
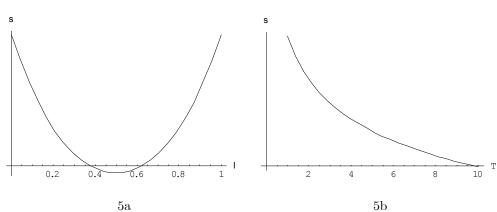


Figure 4 seems somewhat similar to Figure 2, but there are some significant differences owing to a very different specification of the elasticity function. It is again clear that higher transport costs in addition to other factors show a tendency towards driving the dispersion of manufacturing activity across space,

but the allocation of manufacturing labour no longer closely follows the one presented in FKV (which assumes only perfect labour mobility and no intermediate goods). With high trade costs (dashed line) there are five possible equilibrium labour shares, whereby in this case three of those are actually stable, with, in addition to complete agglomeration equilibria, the symmetric equilibrium also being stable ¹⁶. Intermediate and low trade costs still lead to a core-periphery pattern of factor allocation similar to the constant elasticity of substitution version of the model. As before the elasticity of substitution is highest in the two cases with complete agglomeration (which can be seen from Figure 5a), while the elasticity of substitution is, again as before, decreasing with growing trade costs (Figure 5b).

Figure 5: Response of σ in the model to the changes in the share of manufacturing



labour and changes in trade trade costs

Higher trade costs again tend to force a dispersion of manufacturing activity, which leads to a smaller total number of firms operating in the market and sub-

¹⁶As confirmed by the negative slope of $\omega 1 - \omega 2$ curve at the symmetric equilibrium.

sequently smaller elasticity of substitution between varieties. The dispersion effects that prevailed under high transport costs in Figure 2 also appear under the alternative specification of the elasticity function (16). Under the more general elasticity specification (16) the dispersion force caused by the increased elasticity of substitution turns out to be somewhat weaker than in the previous, logarithmic, specification (15), which allows for a slightly stronger possibility of a core-periphery pattern developing. Intermediate and low level transport costs, namely, still lead to an agglomeration of economic activity in one of the regions (whereby in the case of the logarithmic specification only low transport costs ensured the concentration of manufacturing activity). In the latter elasticity specification (16) the increment to the elasticity of an additional firm is smaller than the one observed in the logarithmic specification (15). Although both alternative elasticity specifications enduce a dispersion of economic activity, the dynamics of the adjustment process crucially depend on which elasticity specification is used in the simulations. Interestingly, only the combined effects of high trade costs and product market crowding serve as a dispersion force intense enough to counteract the strong agglomeration forces of the model presented. The fact that the commodity space becomes gradually crowded serves as a negative externality of agglomeration, whereby none of the new entrants into the market take account of these costs as they decide whether to enter the market or not (product market crowding hence serves as a dispersion force). Competitive market solutions therefore fail to encompass the costs brought forth by the crowding of the product space (those would be taken into account in the social

planner solution).

6 Conclusions

The preceding sections revealed the sensitivity of economic geography models to different specifications of the elasticity function. There are significant differences in the reallocation of labour under different specifications of the elasticity function and the results are certainly not robust to different specifications of the function. The possible effects of commodity space crowding should therefore be seriously considered in economic geography models and the relevance of the assumption of constant elasticity of substitution should rightly be questioned. Product market crowding introduces another dispersion force in the model counteracting strong agglomeration forces represented by intermediate goods in production and a perfectly mobile labour force. When product market crowding is not explicitly included in the model it tends to exhibit strong core-periphery patterns even with high transport costs, which emphasizes the importance that increasing elasticity as a dispersion force can have when firms concentrate in one location. The results however depend crucially on the specification used to describe the elasticity function. The simpler form of the elasticity function (logarithmic function) carries with it a stronger effect of an additional firm on the elasticity of substitution and a stronger incentive for firms not to agglomerate compared with the alternative functional forms. Depending on the modeling of product space crowding endogenous elasticity of substitution turns out to be

a fairly strong dispersion force which could drive the movement of economic activity from the economic centers to the peripheral regions.

7 Appendix A

Derivation of the price elasticity of demand

The approach undertaken in this section follows the one in Yang, Heijdra (1993):

Following FKV, 1999: 48 the uncompensated consumer demand function for manufacturing is

$$m(j) = \mu Y \frac{p(j)^{-\sigma}}{G^{-(\sigma-1)}}$$
 or $m(j) = \mu Y \frac{p(j)^{-\sigma}}{\int_0^n p(i)^{1-\sigma} di}$, (A1)

while the own elasticity of demand is defined as

$$\varepsilon \equiv \frac{\partial m(j)}{\partial p(j)} \frac{p(j)}{m(j)},\tag{A2}$$

hence the price elasticity of demand in the monopolistic competition setting is (totally differentiating (A1) and assuming perfect symmetry)

$$\varepsilon = -\sigma + \frac{\sigma - 1}{n},\tag{A3}$$

since preferences are described by a Cobb-Douglas function and income (Y) is assumed not to depend on price changes¹⁷.

 $^{^{17}}$ Price changes would affect income through the wage setting equation (zero profit condition). This effect could easily be introduced into the analysis, but would not significantly alter the results presented.

8 Appendix B

Derivation of the elasticity of substitution function

In the following section I present the derivation of several possible functional relationships between the elasticity of substitution (σ) or the intensity of the preference for variety in manufacturing (ρ) and the number of varieties (firms) competing in the market (n). The different functional forms depend crucially on the assumptions used in their derivation. At first I will use more restrictive assumptions enabling me to attain simpler versions of the desired functional relationship, but will gradually loosen the restrictions leading to more complicated and more realistic functional forms.

The approach undertaken in this section to derive the functional relationship between ρ and the number of firms in the market will base on combining consumer utility maximization as represented by the price index equation

$$G = \left[\int_0^n p(i)^{\frac{\rho}{\rho-1}} di\right]^{\frac{\rho-1}{\rho}},\tag{B1}$$

whereby G represents the price index of manufacturing products, n is the number of varieties and p the price of a manufacturing variety with producer profit maximization and the zero-profit condition. To obtain the derivative of the price index with respect to the number of varieties the Leibnitz rule of integral differentiation has to be implemented and the resulting total derivative is

$$-\frac{\frac{\partial\rho}{\partial n}}{(1-\rho)^2}\ln G + \frac{\rho}{\rho-1}\frac{\frac{\partial G}{\partial n}}{G} = \frac{1}{\int_0^n p(i)^{\frac{\rho}{\rho-1}}di}\Delta, \quad where$$
(B2)

$$\Delta = \left(\frac{\rho}{\rho-1} \int_0^n p(i)^{\frac{1}{\rho-1}} \frac{\partial p(i)}{\partial n} di + p(n)^{\frac{\rho}{\rho-1}} - \frac{\frac{\partial \rho}{\partial n}}{(1-\rho)^2} \int_0^n p(i)^{\frac{\rho}{\rho-1}} \ln(p(i)) di\right),$$

imposing market symmetry and simplifying the above equation, we get

$$\frac{\partial G}{\partial n}\frac{1}{G} = \frac{\partial p}{\partial n}\frac{1}{p} + \frac{\rho - 1}{\rho}\frac{1}{n} + \frac{\partial \rho}{\partial n}\frac{\ln n}{\rho^2}$$
(B2')

with profit maximizing producer behavior and monopoly pricing, prices are determined by (FKV, 1999: 51)

$$p = \frac{w^{1-\alpha}G^{\alpha}}{(\varepsilon+1)}\varepsilon c = c\frac{\rho-n}{\rho(1-n)}w^{1-\alpha}G^{\alpha},$$
(B3)

whereby the right hand side of the equality comes about due to (A3). The marginal costs in the numerator of (B3) include intermediate goods costs, G. The derivative with respect to the number of varieties is

$$\frac{\partial p}{\partial n}\frac{1}{p} = \frac{\partial c}{\partial n}\frac{1}{c} + \frac{1}{\rho - n}\left[\frac{\partial \rho}{\partial n} - 1\right] + (1 - \alpha)\frac{\partial w}{\partial n}\frac{1}{w} + \alpha\frac{\partial G}{\partial n}\frac{1}{G} - \frac{1}{\rho}\frac{\partial \rho}{\partial n} + \frac{1}{1 - n},$$
(B4)

where the first part of the right hand side turns out to be zero since c is a constant.

The composite index of the consumption of manufactured goods is defined as (FKV, pg.46)

$$M = \left[\int_{0}^{n} m(i)^{\rho} di\right]^{1/\rho},\tag{B5}$$

where M is the composite index of manufacturing consumption and m the consumption of each available variety.

Assumption 1:

Both the composite index of manufacturing good consumption as well as the consumption of each variety of manufacturing goods do not change with the introduction of new varieties in the market.

This assumption may seem counterintuitive given the utility function is of the Spence-Dixit-Stiglitz preference for variety form, which tend to exhibit increases in utility with additional varieties on offer. But that result depends significantly on the assumption of a constant elasticity of substitution. Assumption 1, on the other hand, means that the beneficial effect of new varieties in the market in exactly offset by the increased elasticity of substitution (less differentiation between varieties). With each new variety the product space becomes increasingly crowded decreasing "the variety effect on utility".

From the definition of the composite index of manufacturing consumption (B5) combined with the assumption of perfect symmetry in the market yields

$$M \equiv n^{\frac{1}{\rho}}m,\tag{B6}$$

Given assumption 1 the ratio M/m is a constant (k) that gives

$$\rho = \frac{1}{\ln k} \ln n. \tag{B7}$$

Given assumption 1 and some additional restrictions placed on the value of k, ρ turns out to be a simple logarithmic function of the number of varieties ¹⁸. Assumption 2:

¹⁸In the case where k is fixed, A2 only hold for n between 1 and e given the restrictions imposed on ρ (0 < ρ <1). More generality can be obtained by assuming that the ratio M/m remains larger than n for all n.

The number of firms (varieties) in the market has no effect on the wages set in the labor markets.

Assumption 2 therefore eliminates any possible effects of additional firms on the labor markets by imposing a perfectly elastic labor supply curve. (labor market is big enough to accommodate any demand at a given wage). This assumption combined with the assumption that agricultural prices are not effected by an increase in the number of manufacturing varieties leads to the fact that additional firms do not effect the total income in a location. The above assumption simplifies (B4) but does not offer immediate solutions for the required functional form. The functional relationship can be pinned down by either of the two following assumptions.

Assumption 2a:

A change in the number of available varieties (firms) in the market does not effect the price index of manufacturing goods G.

This assumption implies that despite the fact that new entrants drive down the price of manufacturing goods, they also increase the elasticity of substitution and the two effects cancel out. This resembles closely the assumption of a large number of firms used commonly in applications of the Dixit-Stiglitz approach (Benassy, 1996).

Combining assumptions 2 and 2a yields

$$\frac{\partial \rho}{\partial n} = \frac{(1-\rho)\rho}{n(\ln n - \rho)},\tag{B8}$$

assuming away the integration constant (without loss to generality suppose the

constant were exactly zero)

$$\rho = \rho(1-\rho)\ln(\ln n - \rho), \tag{B9}$$

it can be easily seen that ρ is increasing in n at a decreasing rate. (B9) is positive if $n > e^{\alpha \rho}$, which is satisfied in general if $n \ge 2$.

Assumption 2b:

A change in the number of available varieties in the market does not effect the price of manufacturing goods.

Assumption 2b is complementary to assumption 2a allowing for changes in the price index while holding the prices of individual varieties constant. In this case an increase in the price index will be exactly offset by the increased elasticity of substitution.

Combining assumptions 2 and 2b yields

$$\frac{\partial \rho}{\partial n} = \frac{(1-\rho)\rho}{n(\ln n - \alpha\rho)},\tag{B10}$$

which can be integrated over n to

$$\rho = \rho(1 - \rho) \ln(\ln n - \alpha \rho). \tag{B11}$$

(B11) is very similar to (B9) as expected. Again ρ is an increasing function of n at a decreasing rate, compared to (B9) the rate of increase is greater. The assumptions used above to derive explicit functional relationships between ρ and the number of firms in the market are fairly restrictive and tend to diminish the general usefulness of the above functions. Abandoning those assumptions requires an additional equation in order for the system of equations to be exactly identified. The solution comes in the form of the zero-profit condition for manufacturing firms (FKV, 1999: 243)

$$w\lambda = (1 - \alpha)npq,\tag{B12}$$

whereby I assume a one location world (share of manufacturing labor $\lambda = 1$), which means the derivative of wages with respect to the number of varieties can be determined as

$$\frac{\partial w}{\partial n}\frac{1}{w} = \frac{1}{n} + \frac{\partial p}{\partial n}\frac{1}{p} + \frac{\partial q}{\partial n}\frac{1}{q},\tag{B13}$$

with

$$\frac{\partial q}{\partial n} = \frac{F(\frac{\partial \rho}{\partial n})}{c(1-\rho)^2},\tag{B14}$$

where F are fixed and c unit costs of manufacturing production. (B15) follows from totally differentiating (3). Combining (B2), (B4), (B13) and (B14) yields

$$\frac{\partial\rho}{\partial n} = \frac{\frac{(\rho-\alpha)(\rho-n)(1-n)+\rho n(\rho-1)}{n(1-n)}}{\frac{\rho^2 n - \rho^2 + \alpha \rho^2 - \alpha \rho n - \alpha \ln n \left[\rho(1-\rho) - n(1-\rho)\right]}{\rho(1-\rho)}},$$
(B15)

the sign of (B15) is ambiguous, rewriting it presents a clearer picture of necessary condition for the determination of the effect of the number of firms on the elasticity of substitution.

$$\frac{\partial \rho}{\partial n} = \frac{\frac{(\alpha - \rho)(n - \rho)}{n} + \frac{\rho(1 - \rho)}{(n - 1)}}{\frac{n(\rho - \alpha) + \rho(1 - \alpha)}{(1 - \rho)} + \frac{\alpha \ln n [n - \rho]}{\rho}},$$
(B16)

assuming the number of active firms n is a nonzero integer then $n>\rho$ and n>

 α . The necessary and required conditions for the effect of the number of firms on the elasticity of substitution to be positive are (required condition) $\alpha \ge \rho$ and necessary condition

$$\frac{\alpha(n-\rho)+\rho(1-n)}{(1-\rho)} < \frac{\alpha \ln n \left[n-\rho\right]}{\rho},\tag{B17}$$

if the required condition fails $\alpha < \rho$ then $(\rho - \alpha) < \frac{n\rho(1-\rho)}{(n-1)(n-\rho)}$ has to hold for (B16) to be positive. These conditions are not unlike the "black hole" conditions of FKV, 1999.

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